CHAPTER 10

MATHEMATICS

The following should be read in conjunction with the Mathematics question papers of the November 2017 Examinations.

10.1 PERFORMANCE TRENDS (2014–2017)

The number of candidates who wrote the Mathematics examination in 2017 decreased by 20 809 in comparison to that of 2016. The performance of the candidates in 2017 reflects a slight improvement at the 30% level to 51,9% and at the 40% level to 35,1%.

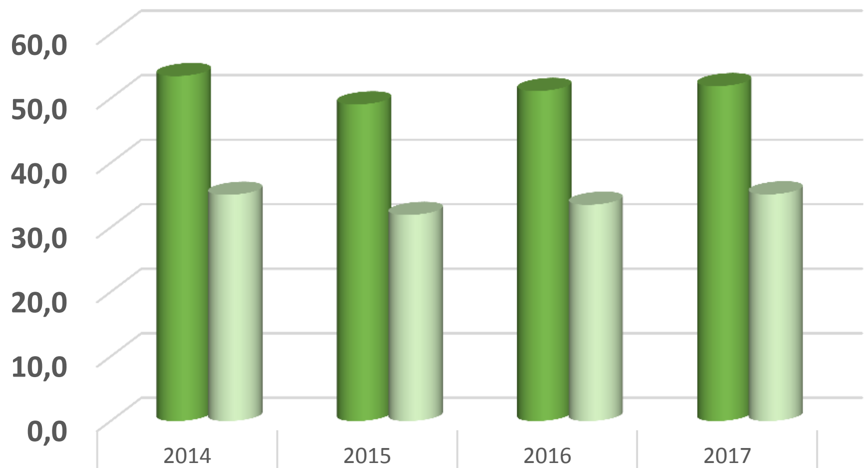
Table 10.1: Overall achievement rates In Mathematics

Year	No. wrote	No. achieved at 30% and above	% achieved at 30% and above	No. achieved at 40% and above	% achieved at 40% and above
2014	225 458	120 523	53,5	79 050	35,1
2015	263 903	129 481	49,1	84 297	31,9
2016	265 912	136 011	51,1	89 119	33,5
2017	245 103	127 197	51,9	86 096	35,1

There has been steady improvement in performance over the last few years, suggesting that there is now some degree of stability in the subject after the introduction of the CAPS curriculum. The increase in the number of candidates who answer the knowledge and routine questions correctly indicate that teachers and candidates are familiar with the manner in which the curriculum will be assessed and the degree of challenge expected in the examination. Also pleasing to note is the improved performance in answering routine questions in the new topics, namely, Probability and Euclidean Geometry.

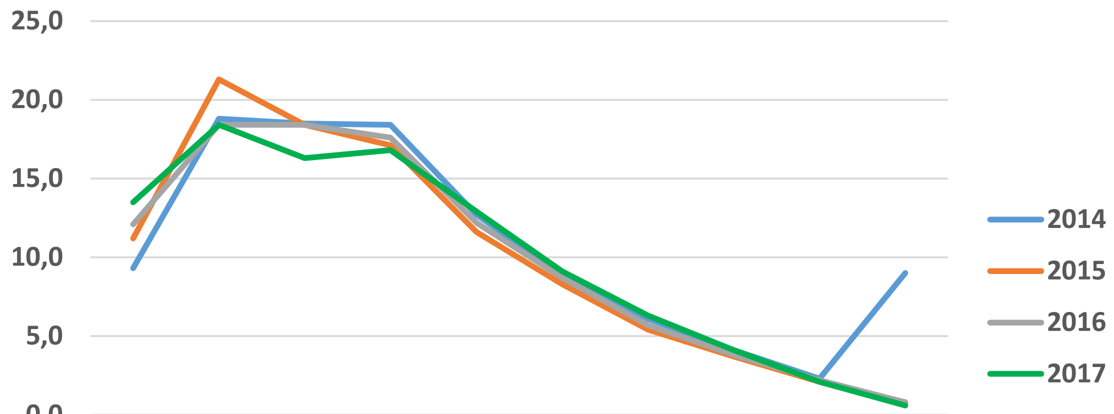
Performance will be further enhanced if attention is given to the following areas: strengthening the content knowledge in Trigonometry and learners' exposure to complex and problems solving type questions. Learners need to be exposed to complex questions and problem solving across all topics in the curriculum. This should start in earlier grades.

Graph 10.1.1: Overall achievement rates in Mathematics (percentage)



	2014	2015	2016	2017
% achieved at 30% and above	53,5	49,1	51,1	51,9
% achieved at 40% and above	35,1	31,9	33,5	35,1

Graph 10.1.2: Overall achievement rates In Mathematics (percentage)



	0-9	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-100
2014	9,3	18,8	18,5	18,4	12,7	8,9	6,1	4,1	2,3	9,0
2015	11,2	21,3	18,4	17,1	11,6	8,3	5,4	3,7	2,1	0,8
2016	12,1	18,4	18,4	17,6	12,2	8,7	5,7	3,8	2,2	0,8
2017	13,5	18,4	16,3	16,8	12,9	9,1	6,3	4,1	2,1	0,6

10.2 OVERVIEW OF LEARNER PERFORMANCE IN PAPER 1

(a) Candidates' performance was marginally better than in 2016. This was evident from the marking process where more candidates managed to pass and get some marks in majority of the questions. It is encouraging to note that many candidates were better prepared to answer the routine questions.

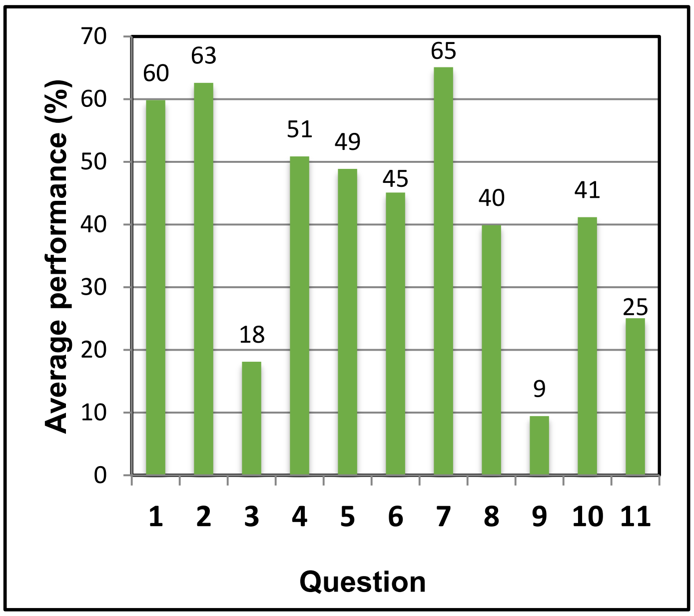
(b) The algebraic skills of the candidates are poor. Most candidates lacked fundamental and basic mathematical competencies which could have been acquired in the lower grades.

(c) Whilst calculations and performing well-known routine procedures form the basis of answering questions in a Mathematics paper, deeper understanding of definitions and concepts cannot be overlooked.

10.3 DIAGNOSTIC QUESTION ANALYSIS FOR PAPER 1

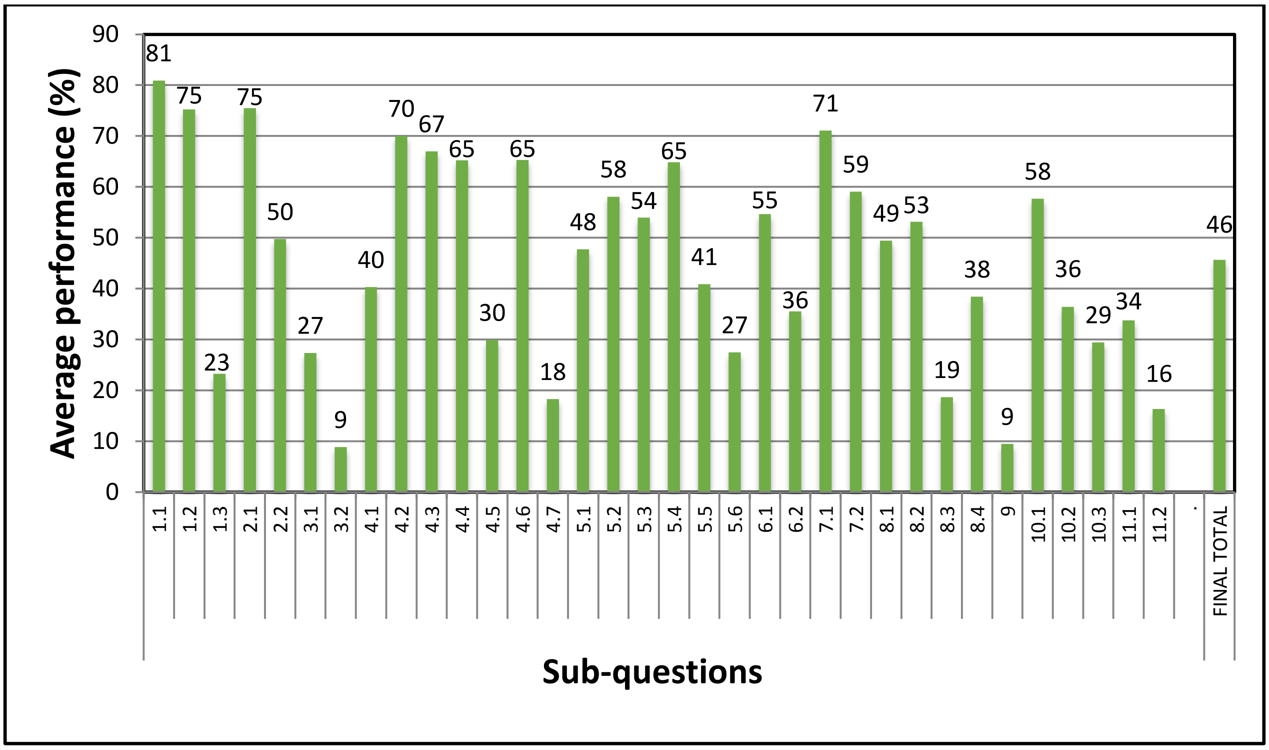
The following graph is based on data from a random sample of candidates. While this graph might not accurately reflect national averages, it is useful in assessing the relative degrees of challenge of each question as experienced by candidates.

Figure 10.3.1 Average percentage performance per question for Paper 1



Q1	Equations, Inequalities and Algebraic Manipulation
Q2	Number Patterns & Sequences
Q3	Number Patterns & Sequences
Q4	Functions and Graphs
Q5	Functions and Graphs
Q6	Finance
Q7	Calculus
Q8	Calculus
Q9	Calculus – Application in optimisation
Q10	Probability
Q11	Counting Principles

Figure 10.3.2 Average percentage performance per sub question for Paper 1



10.4 ANALYSIS OF LEARNER PERFORMANCE IN EACH QUESTION IN PAPER 1

QUESTION 1: ALGEBRA

Common errors and misconceptions

(a) Writing down the quadratic formula correctly and correct substitution therein remain problematic among some candidates. Some candidates wrote the quadratic formula incorrectly, for example, $x = \dfrac{b \pm \sqrt{b^2 - 4ac}}{2a}$ or $x = \dfrac{-b \pm \sqrt{b^2 + 4ac}}{2a}$ or $x = -b \pm \dfrac{\sqrt{b^2 + 4ac}}{2a}$. Such candidates lost marks for an incorrect formula. Incorrect rounding still poses a problem.

(b) In Q1.1.3, candidates did not square the RHS of the equation correctly and arrived at the following: $x^2 - 5 = 2x$ or $x^2 - 5 = 2x^2$ or $x^2 - 5 = 4$. In general, many candidates neglected to check their answers when solving surd equations.

(c) In Q1.2, some candidates wrote $-y = 4 - 3x$, and then substituted y with $4 - 3x$. Many candidates made mistakes in simplifying $-(3x - 4)^2$.

(d) In Q1.3.1, many candidates were able to factorise the expression but could not solve the inequality. Many treated the inequality as an equation. This led to them writing answers that do not make sense: $x > -4$ or $x > -4$ or $4 < x > -4$. Candidates also showed little or no understanding of the set builder or interval notation.

(e) The use of the words 'and' and 'or' are not understood.

(f) Candidates had very little idea on how to answer Q1.3.2. The mention of 'negative roots' resulted in many candidates making the assumption that $\Delta < 0$ which resulted in a breakdown. Many candidates opted for the algebraic route and that made the solution much more difficult. Very few used the much shorter and more direct graphical route.

Suggestions for improvement

(a) Constant practise of fundamentals cannot be over-emphasised. Whilst a calculator may provide learners with the answers to a question, it is imperative that the learners understand the background to these answers. Answers, on their own, are meaningless in the context of concepts.

(b) Teachers should teach factorisation intensively. Writing a quadratic equation in standard form means to make the RHS of the equation equal to 0.

(c) When dealing with surd equations, learners should be reminded that we need to square both sides of the equation in order for the balance to maintain. We do not only square the radical parts of the equation. Teachers must emphasise that implicit restrictions are placed on surd equations and learners should continue to test whether their answers satisfy the original equation. Learners must master this concept in Grade 11.

(d) In teaching inequalities, integrate algebra with functions so that learners have a visual understanding of inequalities. Stress the meaning of the inequality signs in the teaching of both algebra and functions. Demonstrate different methods to solve inequality problems so that learners can choose the method they understand best.

(e) Teachers should explain the difference between *and* and *or* in the context of inequalities. Learners cannot use these words interchangeably as they are very different in meaning.

(f) Rounding off should be clearly understood by learners and rounding off instructions should be emphasised in class-based assessments. Teachers are advised not to condone errors due to rounding in school-based assessment tasks.

(g) Teachers should explain the meaning of roots of an equation and show the graphical representation of the different scenarios.

QUESTION 2: PATTERNS

Common errors and misconceptions

(a) Candidates could not subtract two negative numbers correctly. This resulted in them obtaining a positive second difference instead of the required negative value. Some candidates wrote down the next three terms as the answer to Q2.1.1. This was not required.

(b) Some candidates calculated b = 0 but then wrote the general term as $T_n = -3n^2 + n + 8$ instead of $T_n = -3n^2 + 8$. In spite of the question stating that number pattern is quadratic, some candidates wrote the general term as being linear: $T_n = -3n + 8$.

(c) Some candidates regarded the given value -25939 as n instead of T_n. Many candidates accepted n = -93 as a solution to Q2.1.3. This shows that they have little understanding of the value of n in the context of number patterns.

(d) Many candidates made mistakes in Q2.2.1 because they did not use brackets, i.e. they wrote $k + 8 - 2k - 7 = 2k - 1 - k + 8$ instead of $k + 8 - (2k - 7) = 2k - 1 - (k + 8)$. They obtained an answer for k that did not satisfy the requirements of an arithmetic sequence and resulted in a breakdown. Some candidates treated the sequence as geometric instead of arithmetic and applied $\frac{T_2}{T_1} = \frac{T_3}{T_2}$.

(e) Many candidates failed to extract the sequence comprising of only even terms but proceeded to calculate the sum of the first 30 terms of the original sequence.

Suggestions for improvement

(a) While teaching this section, teachers should emphasise the difference between the *position* and the *value* of a term in a sequence. Learners must read the questions carefully so that they do not confuse what is required of them.

(b) Learners need to analyse the type of sequence they are working with and which formulae are applicable to it.

(c) Teachers need to expose learners to patterns where other patterns need to be created within the given pattern. Teachers are encouraged to formulate their own questions and include them in their informal and formal tasks. This will encourage learners to think logically.

(d) Teachers need to expose learners to number patterns in which terms include variables and not only numeric values. Learners should establish a pattern on their own if given diagrams or pictures.

(e) Teachers should use the correct notation and mathematical language on a daily basis in the classroom. Encourage learners to speak the mathematical language in the classroom.

Teachers also need to realise that learners' understanding of the concepts is more important than them merely doing routine procedures in a section.

QUESTION 3: PATTERNS

Common errors and misconceptions

(a) Many candidates did not know how to approach Q3.1. Some correctly wrote that $a + ar = 2$ but could not proceed further. Some candidates used the sum formula and this complicated matters even further.

(b) Most candidates found difficulty in engaging with Q3.2. Many candidates could not interpret the given sigma notation, i.e. $\sum_{n=3}^{\infty} T_n = \frac{1}{4}$, which implied that the sum to infinity from the third term onward equalled $\frac{1}{4}$. Candidates mistakenly used $T_n = \frac{1}{4}$ or $\frac{a}{1-r} = \frac{1}{4}$, resulting in a breakdown. Although the question specified that the sequence comprised only positive terms, candidates failed to exclude negative values of *a* and *r* as solutions.

Suggestions for improvement

(a) Attention needs to be paid to the basics in number patterns. The concept of sum of terms needs to be explained. This topic is not merely using a formula to obtain an answer but it requires a deeper understanding of concepts.

(b) Teachers need to clarify that sigma notation is a short-hand notation of a series of terms and that this does not always include the first term of a series.

(c) Expose learners to 'unseen' type questions where unfamiliar patterns are formed. Convince them that these are generally easy to solve.

(d) Teachers should consult the array of different questions in sequences, as tested in the last 9 years. Contextual examples where reading is required should be emphasised.

QUESTION 4: FUNCTIONS (PARABOLA AND STRAIGHT LINE)

Common errors and misconceptions

(a) In Q4.1, candidates did not realise that they needed to solve for *a* and *b* by using simultaneous equations. The integration of calculus contributed to poor performance. Candidates who made the connection with calculus then equated the derivative to zero when $x = -1$, in other words, they assumed that the given point was a turning point.

(b) Some candidates used the given values of *a* and *b* to show that $a = 0,5$ and $b = 2$.

(c) Although the values of *a* and *b* were given in Q4.1, candidates used other values for *a* and *b* in Q4.2. Some candidates used $f(x) = \frac{1}{2}x^2 + 2x + 6$ due to the confusion of the negative sign in $f(x) = -ax^2 + bx + 6$.

(d) Some candidates calculated the turning point to be (2 ; 6) and this conflicted with the fact that the *y*-intercept is (0 ; 6). In Q4.4, some candidates sketched a cubic function instead of a parabola. In Q4.6, some candidates drew an exponential curve instead of a straight line.

(e) Q4.5 and Q4.7 were higher-order questions that involved reading off intervals from the graph(s). However, many candidates attempted to answer these questions by algebraic manipulation. This approach only compounded matters: not only did they have to deal with complex inequalities, but they spent a lot of time in answering using this method.

Suggestions for improvement

(a) Learners should be exposed to questions that integrate different topics in class-based assessment tasks. In this way, they will move away from the idea that topics must be learnt in isolation.

(b) Where the values of coefficients are given in a question, learners should be informed that they must use these values in subsequent questions. In general, learners should be trained that if they fail to prove certain values in a question they are allowed to use these values in subsequent questions.

(c) Emphasise the drawing of graphs as well as the interpretation of functions.

(d) It is important for teachers to emphasise that learners must do thorough revision of Grade 11 work on Functions. Learners have to know and understand the basic shapes, properties etc. of each type of graph very well before they can go on to more difficult questions.

(e) Learners need to be made aware that algebraic manipulation is not the only method to solve questions. Some questions can be solved more efficiently with the use of graphs.

QUESTION 5: FUNCTIONS (HYPERBOLA, LOGARITHMIC GRAPH AND INVERSES)

Common errors and misconceptions

(a) Most candidates could not state the range of the hyperbola. Candidates were unable to differentiate between $y \in R$ and $y \in R$, $y \neq -1$. Some gave the answer in terms of *x*, i.e. these candidates confused the domain with the range.

(b) Candidates did not use the fact that OB = BE and could not establish the vertical asymptote of the hyperbola. Many candidates gave the equation as $y = \frac{2}{x+2} - 1$ instead of $y = \frac{2}{x-2} - 1$ as it should be with an asymptote of $x = 2$.

(c) Whilst candidates may have used the accuracy of the sketch to intuitively write down the answer for t, they were expected to show their working steps. Candidates opting to use the function for g had greater difficulty in arriving at the answer than those who used the function for f. Some candidates obtained a negative value for t but could not see that this answer could not have made sense. Some candidates tried to solve $\frac{2}{x-2} - 1 = \log_3 x$ but were unsuccessful.

(d) Some candidates did not realise that $f^{-1}(x) < 3$ means that the y-value of the inverse graph has to be less than 3 (below 3 on the sketch). Many candidates presented the incorrect answer of $x \geq 1$.

(e) Few candidates were able to determine the equation of the axis of symmetry. Some of those who were able to determine the equation of the axis of symmetry, did not realise that the x-intercept of the axis of symmetry passes through B. Very few candidates used the fact that OB=BE in solving for the answer. A number of candidates tried to solve the two equations simultaneously.

Suggestions for improvement

(a) The focus of teaching functions should not only be on sketching the graph. The characteristics and features are also important aspects of a graph. Transformations and interpretation of graphs should also be covered in some depth.

(b) Learners need to get enough practice in determining the equations and drawing the graphs of the inverses of the prescribed functions. Emphasis should be placed on the fact that the inverse of a function is the reflection of the original function about the line $y = x$.

(c) More time must be spent on the meaning of inequalities, what they represent and how to write a domain or range in the correct notation. This includes the emphasis on interval notation and using the inequality signs to represent the same interval.

(d) Teachers should expose learners to questions involving two graphs on one system of axes, incorporating interpretation questions.

QUESTION 6: FINANCE

Common errors and misconceptions

(a) Candidates got confused because they had to work in months. Many struggled to use the correct value of n and divide r by 12. Furthermore, calculating $\sqrt[36]{\frac{12146,72}{10000}}$ was a huge challenge. Some candidates even introduced logarithms in this question. Many candidates used an inappropriate formula e.g. $A = P(1-i)^n$, $F = \frac{x[(1+i)^n - 1]}{i}$ or $P = \frac{x[1-(1+i)^{-n}]}{i}$. Some swopped the values of A and P.

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(b) In Q6.2.1, candidates were not aware of the standard terms of loan repayment and therefore treated the question as if payment was deferred by one month. Candidates often multiplied 54 by 12 and hence used n = 628. Some candidates lost a mark for using the calculator incorrectly.

(c) Most candidates had no clue on how to approach Q6.2.2 and consequently did not attempt this question. Some calculated the rate of interest rather than the amount of interest.

Suggestions for improvement

(a) Finance should be taught with more insight and not merely the substitution of values into a formula only.

(b) Learners need deeper insight into the relevance of each of the formulae and under which circumstances it can be used. More practice in Financial Mathematics is necessary so that learners can distinguish between the different formulae.

(c) Financial Mathematics requires two crucial skills which are often neglected by learners. These are reading skills and calculator skills. The learners must read the Financial Mathematics question very carefully and make sure that they understand what is asked. Calculator work is essential when doing financial maths and this should be practised.

(d) The use of correct English in the teaching of Financial Mathematics is essential.

QUESTION 7: CALCULUS

Common errors and misconceptions

(a) In Q7.1, candidates made simplification or notational errors.

Many candidates made the following notational errors: $f(x)=\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$ or $\lim\limits_{h\to 0}=\frac{f(x+h)-f(x)}{h}$ or $\frac{\lim\ f(x+h)-f(x)}{h\to 0\quad h}$. Some candidates made incorrect substitution for $f(x+h)$. They wrote $f(x+h)$ as $2(x+h)^2-x$ instead of $2(x+h)^2-(x+h)$. Other errors were made in the factorisation of the numerator: $4xh+2h^2-h=h(4x+2h)$ instead of $h(4x+2h-1)$.

(b) The most common error in Q7.2 was when to leave out the derivative notation. Candidates wrote $D_x(3x^2-4x-7)=D_x(6x-4)$ instead of $D_x(3x^2-4x-7)=6x-4$

(c) Candidates struggled to convert from surd form to exponential form: $\sqrt{x^3}$ was written as $x^{\frac{1}{3}}$ instead of $x^{\frac{3}{2}}$. Candidates failed to recognise that $\frac{1}{2}\pi$ is a constant and its derivative is equal to zero.

Suggestions for improvement

(a) Emphasis should be placed on the use of correct notation when determining the derivative, either when using first principles or the rules.

(b) Revision of basic algebraic manipulation is essential in ensuring that learners simplify expressions competently in Grade 12.

(c) Basic exponential laws should be taught properly in earlier grades and revised continuously in Grades 11 and 12.

QUESTION 8: CALCULUS (GRAPHICAL APPLICATION)

Common errors and misconceptions

(a) Candidates did not realise that there must be a change in concavity in the neighbourhood of a point if such a point is to be regarded as a point of inflection. Many merely provided the following answer: $f''(x) = 6x - 12 = 0$ and therefore $x = 2$. This was insufficient to show that a point of inflection exists at $x = 2$.

(b) Candidates did not realise that information about the critical values of the graph were given in the question. Many of them went about calculating these critical values again before sketching the graph. Some candidates joined the points with straight lines instead of curves.

(c) The concept of concavity and the interval for which a function is *concave up* or *concave down* on is not understood well by candidates. Many did not realise that they had to answer about the concavity of the reflection of f in the x-axis.

(d) Candidates tried to answer this question by performing algebraic manipulation instead of transforming the graph. This approach was not only tedious but many candidates made errors along the way and could not answer the question correctly.

Suggestions for improvement

(a) In teaching any function, teachers should expose learners to all aspects of the function. This includes sketching, interpretation of the equation and the graph, as well as finding the equation from given information and transformations. The teaching should also include concepts such as roots, points of intersection, intervals where graphs are relative to one another under a given condition, gradients and equations of tangents.

(b) Learners should be taught to distinguish between the function and its gradient, in other words, the difference between the meanings of $f(x)$ and $f'(x)$.

(c) Expose learners to higher-order thinking questions and interpretation of graphs. Initially teachers should assist learners in understanding what is being asked, what it looks like on the picture and which x-values are relevant to the interval required in the solution.

(d) Teachers must be aware that concavity of functions is explicitly mentioned in *CAPS*. It is important for teachers to discuss the concepts of *concave up*, *concave down* and the intervals for which these occur. The concavity of a graph should also be linked to the second derivative. If a function is concave up on an interval then $f''(x) > 0$ and if a function is concave down on an interval then $f''(x) < 0$.

(e) In a cubic graph, the concavity always changes at the point of inflection. This, however, is not true for all functions, for example $f(x) = x^4$. It is therefore necessary that learners must demonstrate the difference in concavity on either side of the point of inflection (the change of sign in the second derivative).

(f) The cubic function in relation to its first and second derivatives or the quadratic function in relation to its first derivative needs to be taught with great insight. Learners who understood the connection between the function, the first derivative and the second derivative answered this question well.

QUESTION 9: CALCULUS (OPTIMISATION APPLICATION)

Common errors and misconceptions

(a) Many candidates did not see this question as an optimisation question. The most common incorrect answer was that BP = 1 unit. The other common mistake was to take random points for P and to calculate the length of BP.

(b) Few candidates managed to establish that $\text{BP}^2 = x^4 - x^2 + 1$ *or* $\text{BP} = \sqrt{x^4 - x^2 + 1}$ but could not differentiate these expressions. Candidates did not realise that BP is a minimum if BP^2 is a minimum. The majority of these candidates then used trial and error to establish the minimum distance of BP.

Suggestions for improvement

(a) Optimisation should not only be seen in the context of measurements, learners also need to be exposed to optimisation of functions.

(b) This section of calculus is often taught towards the end of the year and therefore learners do not get enough opportunity to practise. Teachers should ensure that there is enough time for learners to understand the application fully.

QUESTION 10: PROBABILITY

Common errors and misconceptions

(a) Some candidates could not interpret the given information correctly. For example. they did not realise that the 12 learners using Instagram and Twitter should be split up between two parts of the diagram, with 8 in the one area and 4 in the other. Also amongst the 61 learners using Instagram, there were some who used some of the other applications as well.

(b) Some candidates took the intersection of the three events to be x.

(c) In Q10.2, many candidates could not set up the correct equation because they omitted the 14 learners who used none of the applications. Some candidates obtained answers that were negative numbers or fractions.

(d) Candidates included parts of the intersection, even though 'ONE of the applications' was emphasised.

Suggestions for improvement

(a) An in-depth explanation of the Venn diagram will clarify the number of events that occur in the various sections, in particular where exactly 1 event occurs, where two events occur simultaneously and where three events occur simultaneously.

QUESTION 11: COUNTING PRINCIPLE

Common errors and misconceptions

(a) The language in this question could have been a barrier to the candidates. Candidates did not read the question carefully. Some used all 26 letters in the alphabet while others excluded 0 as a digit.

(b) Candidates did not know the difference between 5 x 5 and 5! x 5!

(c) Many candidates did not have any idea of how to solve Q11.2. They had difficulty in performing the reverse calculations.

Suggestions for improvement

(a) The section on the fundamental counting principle needs to be taught as clearly and simply as possible. Steer away from formulae and reasoning out scenarios, using diagrams where needed.

(b) The difference between *repeating* and *not repeating* should be explained to learners.

10.5 OVERVIEW OF LEARNER PERFORMANCE IN PAPER 2

(a) Individual performance in the paper varied from very poor to excellent. However, there seems to be a slight improvement in the overall performance in this paper.

(b) Candidates performed well in data handling and analytical geometry.

(c) It seems like candidates are struggling with trigonometry and Euclidean geometry. The number of candidates who did not attempt these questions is a cause for concern. It is encouraging to note that some candidates are making progress, albeit limited, in answering questions on Euclidean geometry.

(d) Integration of topics is still a challenge to many candidates. Mathematics cannot be studied in compartments and it is expected that candidates must be able to apply knowledge from one section to another section of work.

(e) It is evident that many of the errors made by candidates in answering this paper have their origins in a poor understanding of the basics and foundational competencies taught in the earlier grades.

(f) Candidates struggled with concepts in the curriculum that required deeper conceptual understanding. Questions where candidates had to interpret information or provide justification, presented the greatest challenge.

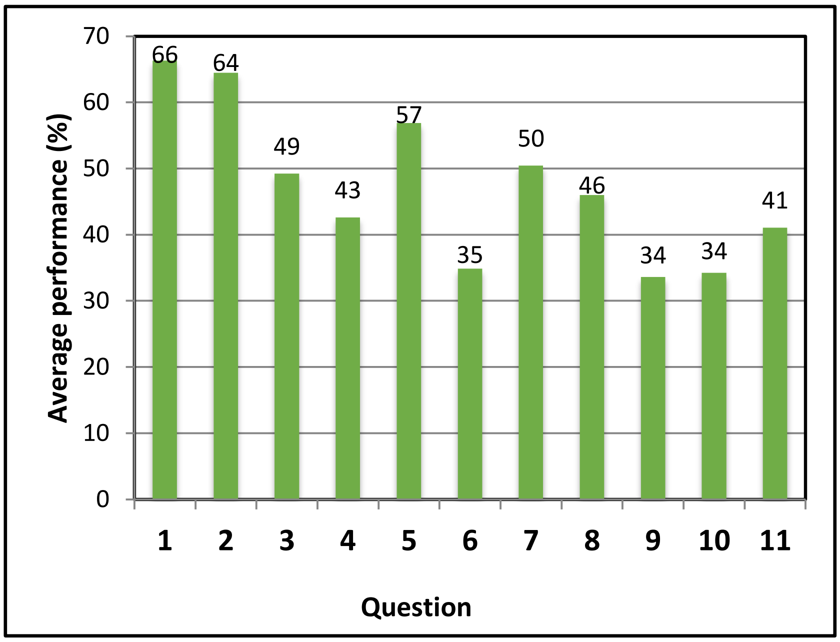
(g) In general, candidates need to exercise caution with algebraic manipulation skills since overlooking certain basic principles or practices results in the unnecessary loss of marks. Although the calculator is an effective and necessary tool in Mathematics, learners appear to believe that the calculator provides the answer to all their problems. Some candidates need to realise that conceptual development and algebraic manipulation are often impeded as a result of the dependence on the calculator.

(h) Candidates need to read the questions with due diligence. By glossing over questions, candidates are overlooking the critical information about the questions.

10.6 DIAGNOSTIC QUESTION ANALYSIS FOR PAPER 2

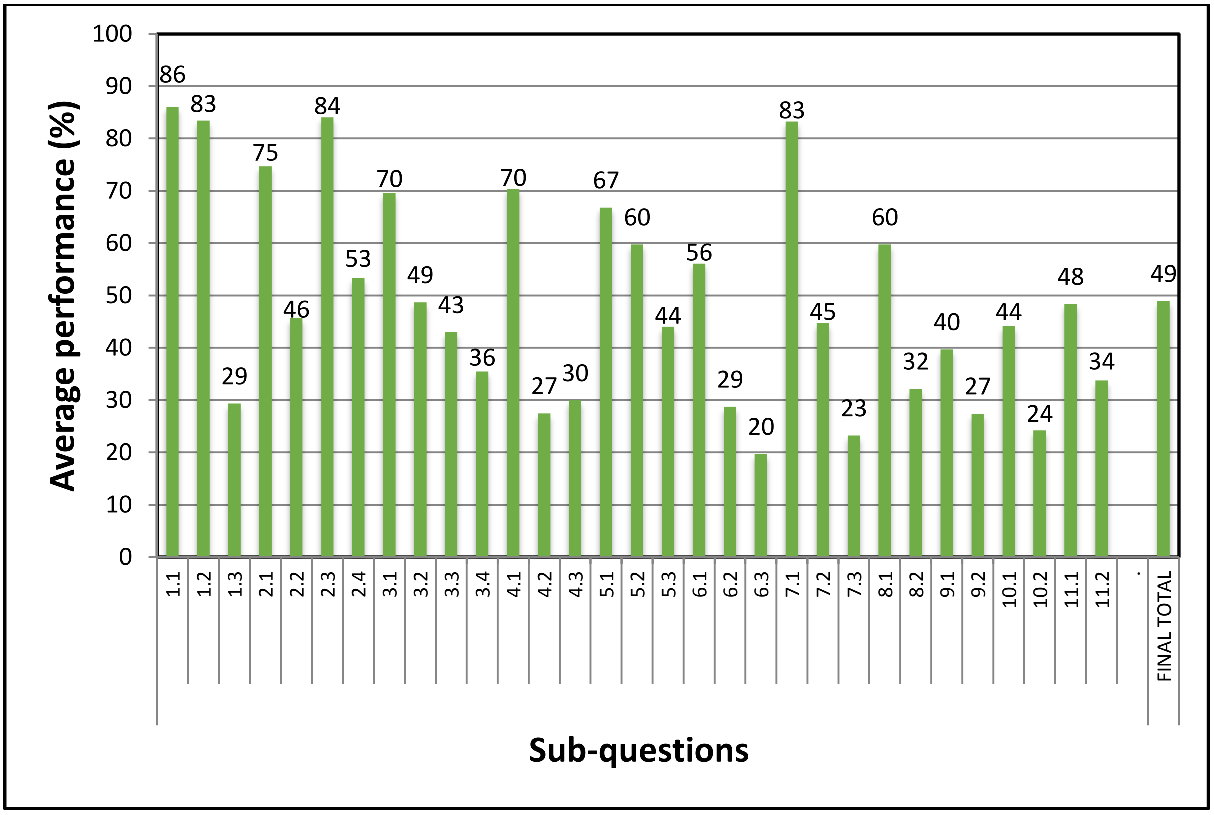
The following graph was based on data from a random sample of candidates. While this graph might not accurately reflect national averages, it is useful in assessing the relative degrees of challenge of each question as experienced by candidates.

Figure 10.6.1 Average percentage performance per question for Paper 2



Q1	Data Handling
Q2	Data Handling
Q3	Analytical Geometry
Q4	Analytical Geometry
Q5	Trigonometry
Q6	Trigonometry
Q7	Trigonometry
Q8	Euclidean Geometry
Q9	Euclidean Geometry
Q10	Euclidean Geometry
Q11	Euclidean Geometry

Figure 10.6.2 Average percentage performance per sub question for Paper 2



10.7 ANALYSIS OF LEARNER PERFORMANCE IN EACH QUESTION IN PAPER 2

QUESTION 1: DATA HANDLING

This question was answered fairly well by most of the candidates. It was disappointing to note that candidates are still unable to round off their answers correctly.

Common errors and misconceptions

(a) In Q1.1, many candidates were able to calculate the values of *a* and *b* correctly but then interchanged these values in the equation $y = a + bx$. A fair number of candidates made errors when rounding. Some candidates selected two points in the data set and applied the algebraic method of calculating the equation of a straight line. This is not acceptable.

(b) Some candidates substituted 11,7 for y instead of x. Others read off from the graph in spite of the question stating that they should use the equation of the least squares regression line.

(c) In Q1.3, candidates merely wrote "increase" as the answer without any motivation or provided incorrect motivation. Others stated that 7,6 is an outlier. Many candidates did not comment on the impact that the new point will have on the regression line.

Suggestions for improvement

(a) Teachers and learners need to be aware that there is a marked difference between the line of best fit and the least squares regression line. The least squares regression line is the line such that the sum of the vertical distances between the data points and the line is a minimum. Calculating the equation of the straight line between two random points in the data set does not meet this criterion and is therefore unacceptable.

(b) Learners should be given multiple opportunities to practice calculator skills. Learners need to be made aware that the operation procedure varies from one brand of calculator to the next. It is in their interest to use the same brand regularly.

(c) Data Handling is not confined to performing certain routine calculations. Making predictions is an important aspect of this topic. Learners must be challenged with "What if …" type of questions in class. Such questions will force learners to think critically and creatively in response to a situation.

QUESTION 2: DATA HANDLING

Candidates' performance in this question was good.

Common errors and misconceptions

(a) In Q2.1.1, candidates spent a lot of time adding the individual values and then divided by 30 instead of 23.

(b) A fair number of candidates calculated the range or semi-interquartile range instead of the inter-quartile range as required in Q2.1.2.

(c) Many candidates calculated the number of girls that were within one standard deviation of the mean. They did not realise that the question required the number of girls that took longer than one standard deviation of the mean.

(d) Some candidates did not understand Q2.4.2. They focussed on the number of boys whose times were between the minimum and the lower quartile. Many ignored the data for the girls in response to this question.

Suggestions for improvement

(a) Teachers must use correct statistical vocabulary and terminology. These concepts must be explained in detail before so that learners may to relate to them. Learners must be made aware that range, inter-quartile range and semi-interquartile range are different in meaning.

(b) Exposure to questions of an interpretive nature cannot be over-emphasised. This should form an integral part of the teaching and learning of this topic. Learners are encouraged to answer different types of questions in data handling to improve their performance.

QUESTION 3: ANALYTICAL GEOMETRY

Common errors and misconceptions

(a) The most common error was using the coordinates of A that was given in Q3.2 in the calculation of the gradient. Some candidates used the coordinates of B and C to calculate the gradient of AC. Candidates made errors when using the calculator and arrived at the gradient of $-\frac{11}{10}$. Candidates did not realise that A, F and C were collinear points and that the gradient of FC was equal to the gradient of AC.

(b) Some candidates assumed that G was the midpoint of AC. Other candidates calculated the x and y-intercepts of BG instead of the coordinates of G.

(c) In Q3.2, candidates used the given coordinates to show that A was (2 ; 5). Some substituted the coordinates into the equation of AC. These candidates were confused as they were trying to prove that (2 ; 5) lies on AC.

(d) In Q3.3, candidates were able to determine the gradient of BG but were unable to link this answer to the gradient of EF. This was on account of them not recalling the midpoint theorem from Grade 10. Some assumed that E was the midpoint of AB.

(e) The properties of quadrilaterals are still not understood by some candidates. This was evident in Q3.4 as many were not able to use properties of a parallelogram to answer the question. Some candidates made the assumption that EF is perpendicular to AC. Very few candidates realised that translation may be used to determine the coordinates of D.

Suggestions for improvement

(a) To answer analytical geometry well, learners should master the properties of quadrilaterals and triangles. These ideas should be included in class based assessment tasks.

(b) Teachers should first revise work done in earlier grades in a specific topic before starting with the same topic at Grade 12 level. In particular, the equation of a straight line and gradients of parallel and perpendicular lines must be revised in Grade 12.

(c) Learners should be encouraged to show all the steps in the working. Continual practice should remedy the basic errors that learners make.

(d) Learners should refrain from making assumptions about features in a question. These need to be proved first before the results can be used in an answer.

(e) The different topics in Mathematics should be integrated. Learners must be able to establish the connection between Euclidean geometry and analytical geometry.

QUESTION 4: ANALYTICAL GEOMETRY

Common errors and misconceptions

(a) In Q4.1.1, some candidates used the gradient formula incorrectly e.g. $m = \frac{x_2 - x_1}{y_2 - y_1}$ instead of $m = \frac{y_2 - y_1}{x_2 - x_1}$. Others made incorrect substitution into the correct formula.

(b) Some candidates were guessing the coordinates of S in order to calculate the equation of RS.

(c) Some candidates could not determine the coordinates of M, the centre of the circle. Others were confused as to whether they should use r or r^2 in the equation of the circle.

(d) In Q4.1.4, many candidates calculated the angle adjacent to θ. They then used this angle as the needed angle for the area rule instead of $S\hat{M}K$.

(e) Some candidates could not visualise "*t*" as it was not given in the diagram. Candidates could not determine the limits of the values of *t*.

(f) A large number of candidates assumed that SM was the perpendicular height of the triangle. Some candidates assumed that SMK was a right-angled triangle. Other candidates assumed that $S\hat{M}K = 180° - \theta$.

Suggestions for improvement

(a) Teachers need to revise the concept of perpendicular lines and gradients, in particular that the tangent is perpendicular to the radius at the point of contact.

(b) Learners must be taught to refrain from assuming facts that are not given. The order in which learners answer questions is important. You cannot use a result that you have not already shown to be true in an answer.

(c) Teachers should ensure that they expose learners to assessments that integrate Analytical Geometry and Euclidean Geometry.

(d) Algebraic manipulation is required in both Mathematics papers. This is a skill that learners need to master.

QUESTION 5: TRIGONOMETRY

Common errors and misconceptions

(a) In Q5.1, candidates struggled with reduction formulae, especially with the signs of the reduced trigonometric ratios. Some complicated matters by applying compound formulae unnecessarily and made mistakes in the process. Some learners could not apply co-functions correctly.

(b) Some candidates did not realise that the angle is in the fourth quadrant and the *y*-value or *t*-value should be negative, working from $t^2 = (\sqrt{34})^2 - (3)^2$ and $t = \pm 5$ they chose the wrong value $t = 5$ instead of $t = -5$. In Q5.2, incorrect trigonometric ratios were used. In Q5.2.2, the ratio of was mostly incorrectly written as $\frac{3}{5}$ instead of $-\frac{5}{3}$

(c) Incorrect substitution was also a challenge. E.g. in Q5.2.3 $\cos 2\beta = 1 - \sin^2 \beta$ was substituted as $\cos 2\beta = 1 - \sin^2\left(-\frac{5}{\sqrt{34}}\right)^2$ instead of $\cos 2\beta = 1 - \left(-\frac{5}{\sqrt{34}}\right)^2$.

(d) Candidates wrote the expansions for $\cos 2\theta$ and $\sin 2\theta$ incorrectly even though these are given on the formula sheet.

(e) Errors in Q5.3.1 resulted from the incorrect simplification after expanding the compound angles.

(f) In Q5.3.2, the challenge was realising that $\sin 77^0$ and $\sin 43^0$ can be expressed as compound angles that include special angles, e.g. $\sin(60^0 + 17^0) - \sin(60^0 - 17^0)$. Some candidates removed sin as a common factor: $\sin 77^0 - \sin 43^0 = \sin(77^0 - 43^0)$

Suggestions for improvement

(a) Learners need to learn the reduction formulae and know which formula to use in the given situation. They must take cognisance of the quadrants when determining the signs of trigonometric ratios.

(b) Learners must be advised to show all steps when working with reduction formulae. Marks are not awarded to candidates who make errors with the signs.

(c) Learners need exposure on the simplification of expressions containing double and compound angles. Examples should include variables for angles as well as specific angle values. Learners need to be encouraged to use brackets, especially when there is a difference of two trigonometric ratios containing compound angles.

QUESTION 6: TRIGONOMETRY

Common errors and misconceptions

(a) Candidates were unable to draw the graph of y = 2sin$x$ - 1 correctly. Many struggled with the shape as they were unsure of the location of the *x*-intercepts. Some even joined the points with a ruler. Candidates did not observe the domain of the graph and drew arrows at the end of the graph. Candidates did not indicate the co-ordinates of the critical values on the graph.

(b) Very few candidates were able to answer Q6.2 as they did not realise that they needed to use the quadratic formula to solve a trigonometric equation.

(c) Many candidates did not make the link between Q6.2 and Q6.3. As there was no mention of a general solution, candidates were unable to answer this question. Some of those who were able to solve for the value of *x* failed to calculate the *y*-coordinate for the points of intersection.

Suggestions for improvement

(a) When teaching trigonometric graphs, teachers should start first with the original graphs: $y = \sin x$, $y = \cos x$ and $y = \tan x$ using point by point plotting and identify important features of these graphs, then introduce *a*, *p* and *q* as well as their effects.

(b) It is common practice for learners to use calculators to sketch graphs. Hence, they do not pay attention to certain critical features of these graphs. Although learners are expected to produce a sketch graph, there is still a high degree of accuracy required of them.

(c) When discussing the transformation of trigonometric graphs, learners must be alerted to how the critical features and characteristics of the basic graph change for each transformation. In this way, they will be able to visualise the effect of *a*, *p*, *q* and *k* on the basic function.

QUESTION 7: TRIGONOMETRY

Common errors and misconceptions

(a) Candidates had difficulty in seeing the different planes in the sketch.

(b) In Q7.2, candidates failed to link the two right-angled triangles. Further, candidates stated the incorrect trigonometric ratios in these triangles. Candidates often mixed up the sides and angles.

(c) Many candidates were confused with the 3-D orientation of the shape. Some were unable to apply the cosine rule correctly.

(d) Candidates demonstrated poor algebraic manipulation skills when squaring fractions and removing common factors.

Suggestions for improvement

(a) Teachers need to develop strategies to be used when solving right-angled triangles and triangles that are not right-angled.

(b) Unless there is something stated to the contrary, trees, poles, buildings, etc. are assumed to be perpendicular to the ground.

(c) Learners must refer to the formula sheet to ensure that formulae are copied correctly.

(d) In Grades 10 and 11, learners should be exposed to problems that involve a combination of shapes in 2-D. This should develop the skill of identifying common sides and angles in composite shapes.

(e) Learners should be encouraged to highlight the different triangles using different colours. This would allow them to identify the common sides and angles.

(f) Teachers should show learners how to deconstruct composite shapes into several triangles.

(g) Initially, expose learners to numeric questions on solving 3-D problems. This makes it easier for learners to develop strategies on how to solve such questions. Once learners have gained confidence with numeric type questions, they should then be exposed to non-numeric and higher order questions.

QUESTION 8: EUCLIDEAN GEOMETRY

Common errors and misconceptions

(a) Generally, candidates either lost marks for incorrect or incomplete reasons or for naming angles incorrectly.

(b) In Q8.1, many candidates failed to connect $\hat{E}$ to $\hat{D}_2$. They only wrote $\hat{B} = \hat{D}_2 = 50^\circ$ and could not go any further. Many also assumed that CD was parallel to AB and interpreted $\hat{D}_2$ and $\hat{A}$ as alternate angles. Some interpreted $\hat{D}_2$ and $\hat{E}$ as corresponding angles. Some candidates still omit to state the parallel lines when dealing with corresponding angles, alternate angles or co-interior angles. Candidates used the whole of $\hat{C}$ instead of $\hat{C}_2$. Some candidates assumed that F is the centre of the circle.

(c) In Q8.2, many candidates gave the reason as tan-chord theorem instead of the converse of the tan chord theorem.

Suggestions for improvement

(a) Learners need to be informed that merely writing a number of correct statements and reasons will not necessarily earn them marks. The statements must be logical and lead to solving the problem.

(b) Learners should be encouraged to scrutinise the given information and the diagram for clues about which theorems could be used in answering the question. Learners must not make any assumptions about the diagrams as these are not drawn to scale.

(c) Teachers must cover the basic work thoroughly. An explanation of the theorem should be accompanied by showing the relationship in a diagram.

(d) Teachers need to insist that learners name the angles correctly. The fact that learners are naming angles incorrectly at Grade 12 level indicates that this issue has not been dealt with effectively in earlier grades.

(e) Learners should be taught that all statements must be accompanied by reasons. It is important to state which lines are parallel when using corresponding angles, alternate angles and co-interior angles as a reason.

QUESTION 9: EUCLIDEAN GEOMETRY

Common errors and misconceptions

(a) In Q9.1.1, candidates used incorrect ratios to prove that FG || BC. Candidates omitted the reasons or provided incorrect reasons for their statements.

(b) Candidates showed little understanding of the difference between a *theorem* and its *converse*. They did not understand when to use the theorem and when the converse would apply.

(c) In Q9.1.2, candidates could not identify the correct ratios. This was on account of the number of different pairs of parallel lines.

(d) In Q9.2, many candidates confused the ratios with the lengths of the sides in the diagram. Most candidates could not link this question with Q9.1.1.

Suggestions for improvement

(a) Learners should be forced to use acceptable reasons in Euclidean geometry. Teachers should explain the difference between a theorem and its converse. They should also explain the conditions for which theorems are applicable and when the converse will apply.

(b) Diagram analysis must be emphasised.

(c) Learners need to be told that success in answering Euclidean geometry comes from regular practice, starting off with the easy and progressing to the difficult.

QUESTION 10: EUCLIDEAN GEOMETRY

Common errors and misconceptions

(a) Many candidates could not provide the correct reason in Q10.1. They were confused about when to use *perpendicular* and when to use *midpoint* in the statement.

(b) In Q10.2.1, candidates again failed to identify the angles that will result in MN || TS. Some provided a reason that did not correspond with the statement.

(c) In 10.2.2, candidates often made several correct statements but could not prove that TMNS is a cyclic quadrilateral. Again, either the reason for TMNS being a cyclic quadrilateral was missing or the theorem was given instead of the converse.

(d) Many candidates could not make the necessary links to solve Q10.2.3. Majority of the candidates had no idea where to start.

Suggestions for improvement

(a) More time needs to be spent on the teaching of Euclidean geometry in all grades.

(b) Learners need to be told that there is no short-cut to mastering the skills required in answering questions on Euclidean geometry. This requires continuous and deliberate practice.

(c) Learners need to be made aware that writing correct, but irrelevant, statements will not earn them any marks in an examination.

(d) Learners must refrain from making assumptions. If they make a statement about the relationship between sides or angles, then they must prove such a statement as true before they can use it.

(e) The teaching of theorems should be done with the relevant understanding.

QUESTION 11: EUCLIDEAN GEOMETRY

Common errors and misconceptions

(a) Some candidates did not do the constructions or made incorrect constructions but went on to prove the theorem. This was considered a breakdown and these candidates were not awarded any marks.

(b) In Q11.2.1 (b), candidates wrote the reason as *exterior angle of cyclic quadrilateral* instead of its converse.

(c) In 11.2.2(a), candidates assumed that $\hat{D}_2 = \hat{A}_2$. Some used $\hat{D}_2 = \hat{E}$ with the reason tan-chord theorem. This was not the case as BC was the only tangent in the diagram. Some candidates confused similarity with congruency in Q11.2.2(b).

(d) Many candidates could not make the necessary link with Q11.2.2(b) in answering Q11.2.3(a). Some attempted to use the theorem of Pythagoras but were unsuccessful at solving the problem. Candidates did not realise that they needed to use similarity to answer this question.

(e) Many candidates had no idea where to start answering Q11.2.3(b). They did not realise that they needed to use trigonometry to answer this question.

Suggestions for improvement

(a) Learners need to be made aware that writing correct, but irrelevant, statements will not earn them any marks in an examination.

(b) Attention must be paid to reasons. Teachers should not condone the use of incorrect reasons in classwork and class based assessment tasks.

(c) Learners must refrain from making assumptions.

(d) Learners need to be exposed to questions in Euclidean geometry that integrates trigonometry. Learners need to revise their Euclidean geometry throughout the year.