Chapter 2

TECHNICAL MATHEMATICS

The following report should be read in conjunction with the Technical Mathematics question papers of the November 2019 examinations.

2.1 PERFORMANCE TRENDS (2018–2019)

In 2019, 9 670 learners sat for the Technical Mathematics examination. The performance of the candidates in 2019 showed an overall decline in comparison to 2018. The performance at 30% and above was 42,7%, representing a decline of 8,0% from 2018. The performance at 40% and above declined by 7% in 2019 to 24,1%.

Table 2.1.1(a) Overall Achievement Rates in Technical Mathematics

<table>
<thead>
<tr>
<th>Year</th>
<th>No. wrote</th>
<th>No. achieved at 30% and above</th>
<th>% achieved at 30% and above</th>
<th>No. achieved at 40% and above</th>
<th>% achieved at 40% and above</th>
</tr>
</thead>
<tbody>
<tr>
<td>2018</td>
<td>10 025</td>
<td>5 075</td>
<td>50,7</td>
<td>3 178</td>
<td>31,7</td>
</tr>
<tr>
<td>2019</td>
<td>9 670</td>
<td>4 125</td>
<td>42,7</td>
<td>2 330</td>
<td>24,1</td>
</tr>
</tbody>
</table>

The performance of learners in Technical Mathematics in the 2019 examination can be attributed to the subject being new, with many teachers still trying to cope with the demands of the subject.

However, there is still room for improvement in the performance of the candidates if the challenges surrounding problem-solving skills, mathematical skills, conceptual understanding and integration of topics as well as technically related (modelling) aspects are addressed.

Revision of work from earlier grades will play an integral part in improving performance in the subject. As stipulated in the Technical Mathematics CAPS, 'Mathematical modelling is an important focal point of the curriculum' and that 'Real-life technical problems should be incorporated into all sections whenever appropriate.'
2.2 OVERVIEW OF LEARNER PERFORMANCE IN PAPER 1

General Comments

(a) Candidates performed well in Q1, Q4 and Q6. The topics covered in these questions were Algebra, Functions and Calculus. Candidates performed best in solving simultaneous equations, calculating the derivative using first principles and calculating the intercepts of cubic functions.

(b) Candidates performed poorly in the following topics: Nature of Roots, Finance, Growth and Decay, Logarithms, Calculus applications involving interpretation of cubic functions and Integration. Candidates performed very poorly in the Application of Calculus involving optimisation.
(c) Performance in topics taught in earlier grades was poor in comparison to performance in topics done in Grade 12. This was probably due to inadequate time being allocated for revision of work from the earlier grades.

(d) It was observed that higher-order questions, e.g. interpretation of graphs, were either not answered or poorly answered. Questions in which topics were integrated proved to be beyond the reach of most candidates.

(e) Candidates did not read and follow the instructions as stipulated in the question paper.

2.3 DIAGNOSTIC QUESTION ANALYSIS OF PAPER 1

The following graph is based on data from a random sample of candidates. While this graph might not accurately reflect national averages, it is useful in assessing the relative degrees of challenge of each question as experienced by candidates.

Graph 2.3.1 Average Percentage Performance per Question for Paper 1
2.4 ANALYSIS OF LEARNER PERFORMANCE IN EACH QUESTION IN PAPER 1

QUESTION 1: EQUATIONS AND INEQUALITIES (ALGEBRA)

Common Errors and Misconceptions

(a) In Q1.1.1 candidates did not realise that they had to remove a common factor and then factorise the difference of two squares.

(b) Many candidates failed to write the negative root in Q1.1.2.

(c) In Q1.2.1 many candidates could not determine the correct product. This led to an incorrect standard form. Some candidates equated each factor to $\frac{-13}{3}$. Other candidates were unable to write the $\sqrt{-35}$ as a complex number. They left the roots of the equation as $x = \frac{-1 \pm \sqrt{35}}{6}$. 
When answering Q1.2.2, most candidates did not realise that \((4 - x)\) is not the same as \((x - 4)\). Therefore, they did not change the inequality and lost a mark for the incorrect critical value and incorrect notation.

In Q1.3 candidates were unable to square a binomial, simplify correctly and then determine the correct standard form. Some candidates complicated the equation by making \(x\) the subject.

A few candidates divided by \(V\) instead of \(Z\) when making \(I\) the subject of the formula.

Many candidates did not realise that they needed to multiply by the conjugate in Q1.4.2.

In Q1.5 most candidates omitted the base 2.

Suggestions for Improvement

(a) Solution of simultaneous equations, on a regular basis. These skills are required to answer Grade 12 examination papers. Manipulation skills should be reinforced.

(b) Teachers need to emphasise that if an equation is already given in factor form and equated to zero, then learners need to write down only the required roots of the equation.

(c) Teachers should point out the difference between an equation that is already factorised and equated to zero, e.g. \((3x - 5)(x + 2) = 0\) and one that is factorised and not equal to zero, e.g. \((3x - 5)(x + 2) = -13\).

(d) Teachers should be aware that the integration of topics is possible. When teaching solution of the quadratic equations, teachers should expose learners to the fact that non-real roots in the real number system can be written as complex numbers in the complex number system.

(e) Before teaching quadratic inequalities, teachers should first revise linear inequalities and their graphical representations. In teaching inequalities, teachers should integrate Algebra with Functions so that learners have a visual understanding of inequalities. Furthermore, emphasise the correct use and interpretation of \(or\) and \(and\). It should be stressed that multiplying or dividing by a negative number reverses the inequality sign. Demonstrate different methods to solve inequality problems so that learners may choose the method that they understand best.

(f) Technical Mathematics learners should be exposed to different forms of real-life technical problems.

(g) Teachers need to explain the process of rationalising the denominator in which the fraction contains a complex number in the denominator. Teachers should demonstrate, side by side, the results when a complex number is multiplied by itself versus when the same number is multiplied by its conjugate. This will explain why multiplying the denominator by the conjugate is necessary.
Revision of operations involving binary numbers and the conversion from one number system to another should be done in Grade 12. Teachers should emphasise the importance of writing numbers in the correct notation, i.e. for binary numbers the base 2 should be indicated. Teachers should inform learners that the base for the decimal number system is 10 and that we use the decimal number system more than any of the other number systems. It becomes rather cumbersome to write the base 10 each time we write a decimal number. It is for this reason that the base is omitted from decimal numbers. If there is no base indicated for a number, it is taken that it is a decimal number.

**QUESTION 2: NATURE OF ROOTS**

**Common Errors and Misconceptions**

(a) Many candidates assumed a value for \( p \) and substituted the same in their calculations. They did not understand the meaning of ‘undefined’ and ‘zero’ in Q2.1.1 and Q2.1.2, respectively.

(b) In Q2.2 candidates failed to write the correct standard form and consequently they were unable to identify the values of \( a, b \) and \( c \). Many candidates failed to state the conditions for the discriminant for the given nature of roots.

**Suggestions for Improvement**

(a) Teachers should show learners that the discriminant, \( \Delta = b^2 - 4ac \), originates from the quadratic formula \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \). In other words, the quadratic formula could be written as \( x = \frac{-b \pm \sqrt{\Delta}}{2a} \).

The value that appears under the radical sign determines the nature of the roots of the equation. Teachers need to explain that *undefined* and *zero* are very different concepts. Furthermore, teachers need to explain that literal fractions are *undefined* when the denominator is equal to zero and that these fractions are equal to *zero* when the numerator is equal to zero.

(b) Teachers need to expose learners to application questions involving the nature of roots. These should include solving problems where the conditions are given.

**QUESTION 3: EXPONENTS, SURDS, LOGARITHMS AND COMPLEX NUMBERS**

**Common Errors and Misconceptions**

(a) In Q3.1 many candidates could not apply the laws of exponents and convert from surd form to exponential form.

(b) Some candidates were unable to apply the laws of logarithms in Q3.2 and Q3.3. They also had difficulty with converting a decimal fraction to a proper fraction.
(c) In Q3.4.1 many candidates did not follow the instruction ‘write the equation in rectangular form’ but wrote it in $CIS$ form instead. Several candidates omitted the value of $r$ in the calculations.

(d) Most of the candidates used a calculator in Q3.4.2. They ignored the instruction that that stated that the answer should be in ‘simplified surd form’.

(e) In Q3.5 many candidates did not write the value of $m$ and $n$ as required; instead they left the equation at the simplification step.

Suggestions for Improvement

(a) Learners should revise all exponential laws, the definition and all the laws of logarithms done in Grade 11.

(b) Teachers need to strengthen the concept of prime factors and reinforce the method of converting decimal fractions to proper fractions.

(c) Teachers should emphasise that learners must adhere to the given instructions, particularly instructions that pertain to the use of calculators. They should be reminded that if they fail to adhere to this instruction then they stand to lose marks. Teachers are advised to penalise learners in informal and formal assessment tasks should they choose to ignore instructions that are clearly indicated.

QUESTION 4: FUNCTIONS

Common Errors and Misconceptions

(a) In Q4.1.2 some candidates wrote down the equation of the asymptote instead of the $x$-intercepts.

(b) Many candidates could not determine the product correctly. They were unable to arrive at the correct standard form and hence the coordinates of the turning point were incorrect in Q4.1.3.

(c) In Q4.1.4 candidates wrote the equations of the asymptotes in terms of $p$ and $q$ instead of writing them as equations in terms of $x$ and $y$ respectively.

(d) In Q4.1.5 many candidates used an incorrect scale for their graphs and could not identify the shapes of the required graphs.

(e) Some candidates did not recognise that $d$ was an asymptote of $h$.

(f) Candidates did not know how the respond to a question in which the instruction read: ‘Show that …’

(g) In Q4.2.3 candidates could not simplify the exponential function in which the index is 0.

(h) Candidates could not differentiate between the $range$ and the $domain$. They often used an incorrect notation in the answer. They would give the range as a set of elements in $x$ and the domain as a set of elements in $y$. 
(i) Candidates could not write down the equation of the reflected graph.

(j) Interpretation of graphs posed a challenge for many candidates. They did not understand what was required of them when given that a function is less than the other. Most of the candidates used incorrect notations in their answers.

**Suggestions for Improvement**

(a) Teachers should emphasise the difference between the *intercepts* and the *asymptotes* of graphs. They should also explain that, in most cases, the asymptotes are horizontal and vertical lines. Therefore, the equation of the vertical asymptote should be given as $x = \ldots$ and the equation of the horizontal asymptote should be given as $y = \ldots$

(b) Definition and correct notation of domain and range should be thoroughly explained and demonstrated to learners.

(c) Teachers should expose learners to different ways of finding the coordinates of the turning point of the parabola.

(d) Learners need to be advised to use the table method when sketching graphs and to connect all the points as in the table. Shapes and characteristics of graphs should be thoroughly explained and demonstrated to learners.

(e) Learners need to be taught that when the question states ‘Show/Prove that …’, it means calculate (justify by means of mathematically correct steps) what is given and the final answer reached must match what is stated in the question.

(f) Teachers should incorporate transformations when teaching functions and graphs.

(g) More time must be spent on the meaning of inequalities and what they represent. This includes the emphasis on interval notation and the use of inequality signs to represent the same interval.

(h) Teachers should demonstrate the idea of where one graph is less than the other graph and show learners, in a systematic way, how the interval for which one graph is less than the other is read off the $x$-axis.

(i) Teachers should expose learners to questions involving two graphs on one system of axes, incorporating interpretation questions.
QUESTION 5: FINANCE, GROWTH AND DECAY

Common Errors and Misconceptions

(a) In Q5.1.1 many candidates could not read the correct value from the graph.

(b) In Q5.2.2 some candidates could not identify the correct formula. Some candidates swapped the values of A and P in the formula and many could not make i the subject of the formula.

(c) In Q5.2 many candidates could not identify the correct formula or substituted incorrect values in the formula. Some candidates could not write the equation in logarithmic form. Other candidates calculated the period but were unable to make a conclusion.

(d) When answering Q8.3, many candidates could not set up a timeline showing the different compounding periods within the given 8-year period.

Suggestions for Improvement

(a) Teachers should demonstrate how to read off graphs and how to interpret graphs to obtain the required responses.

(b) Teachers need to indicate that in all formulae P represents the initial amount. In the case of depreciation, P represents the initial cost and in the case of growth, P represents the amount initially invested or the original number. In all formulae, A represents the final value of P after some time has elapsed.

(c) Depreciation is a reduction scenario, i.e. an item loses value after it is bought. This implies that the cost price (P) will always be greater than the book value (A). Therefore, teachers need to emphasise that in scenarios that involve depreciation, the value of P will be greater than the value of A. In contrast, growth is an increasing scenario therefore the value of P will be less than the value of A.

(d) Teachers need to show learners how to select the formula that is appropriate for the scenario presented. They should also demonstrate how to change the subject of the formula. It is advised the learners first substitute values in a formula and then change the subject of the formula.

(e) Teachers should first revise the conversion of an exponential equation to a logarithmic equation before learners are taught to calculate the value of n.

(f) Teachers should teach all compounding periods (annually, quarterly, monthly, semi-annually/half-yearly and even daily). It is advisable to use timelines in order to better understand a complex problem involving several investments and withdrawals.

(g) A good understanding of Financial Mathematics is best developed through practice. It is important to expose learners to different kinds of questions so that they identify the appropriate formula relevant to the scenario.
QUESTION 6: CALCULUS

Common Errors and Misconceptions

(a) In determining the derivative using first principles in Q 6.1, some candidates:

- Used the incorrect notation by omitting \( \lim_{h \to 0} \) or by placing the = sign in the incorrect position \( \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \) or by using the incorrect notation \( f(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \).

- Could not substitute correctly in the formula.

- Omitted brackets when substituting \( f(x) \).

(b) In Q6.2.1 candidates failed to realise that \( a^3 \) was a constant and went on to differentiate \( a^3 \) as \( 3a \).

(c) In Q6.2.2 most candidates failed to write the surd in exponential form, i.e. \( \sqrt{x} \) as \( x^{\frac{1}{2}} \). Candidates were unable to determine the product. Some candidates further simplified the expression containing unlike terms in Q6.3.1. They did not realise that unlike terms cannot be added or subtracted. Candidates failed to apply the exponential laws correctly in cases where they multiplied or divided terms involving \( x \).

(d) Generally, candidates were unable to write the correct differentiation notation or simplify the expression to be differentiated. A few candidates integrated the expression given in Q6.3.2 instead of differentiating it.

(e) In Q6.4.2 many candidates misinterpreted the ‘zero daily profit’. They substituted 0 in the expression instead of equating the expression to 0.

(f) Many candidates did not realise that they had to differentiate first to find the rate of change. They substituted 200 in \( p(x) \) instead of \( p'(x) \).

Suggestions for Improvement

(a) Teachers should emphasise that when determining the derivative using first principles, the notation \( \lim_{h \to 0} \) must be written down and should only be left out when writing the final answer, i.e. once the learner has substituted that value of \( h \). The continued practise of simplification of expressions and algebraic fractions is encouraged.
Teachers should explain to learners that the notation \( f'(x) \) means that one is required to differentiate the expression \( f \) with respect to \( x \). Teachers should draw attention to the various notations used to indicate that the derivative of an expression is required. That is, determine:

\[
\frac{dy}{dx} \quad \text{if} \quad y = x^n,
\]

\[
\frac{d}{dx} \left( x^n \right) \quad \text{and} \quad D_x \left( x^n \right) \quad \text{all have the same meaning.}
\]

Differentiation involves working with the exponent. It is therefore advisable that teachers revise the laws of exponents and the application thereof prior to teaching the rules for differentiation.

Teachers should define a derivative in relation to gradient at a point on a curve, gradient of a tangent, rate of change and calculus of motion. If well explained, then learners will understand that whenever they see a question related to rate of change, they need to calculate the derivative first. When required to calculate the rate of change for an instant, learners need to be aware that they need to substitute the given value in the derivative function and not the original function.

QUESTION 7: CUBIC FUNCTION

**Common Errors and Misconceptions**

(a) In Q7.1 candidates did not use the given straight line to calculate the coordinates, as required in the question.

(b) Candidates did not understand how to respond to this question. In Q7.2 most candidates used the function as if it was giving information instead of proving that the function was as given.

(c) In Q7.3 some candidates used the derivative function to calculate the value of \( x \). However, they then re-substituted these values of \( x \) in the derivative function instead of substituting in the original function when determining the \( y \)-coordinates. Some candidates did not equate the derivative to zero. A few candidates managed to get the two correct \( x \)-coordinates of the turning points but failed to identify the correct one for \( R \) and therefore were unable to calculate the \( y \)-coordinate of \( R \). Many of the candidates used the quadratic method of finding the turning point. This is not applicable to cubic functions.

(d) Many candidates did not understand the concept of a tangent to a curve at a point in Q7.4.1.

(e) In Q7.4.2 and Q7.4.3 candidates struggled with the interpretation of graphs. Most candidates could not identify the correct interval where the graph is above the \( x \)-axis, and as a result the notation and critical values were incorrect. They also could not identify the correct interval where the derivative function is decreasing, and as a result the notation and critical values were incorrect. Some candidates who identified the correct interval where the derivative function was decreasing, failed to write the interval using the correct notation.
Suggestions for Improvement

(a) Learners must be taught how to relate the given information to the questions asked.

(b) Learners need to be taught that when a question asks to ‘show that …’, it means to calculate (justify by means of mathematically correct steps) what is given, and the answer must be as required. Teachers should expose learners to a variety of examples where they are required to ‘show’ or ‘prove’ something.

(c) Teachers need to emphasise that the derivative is equal to zero at the turning points. Teachers should indicate to learners that \( x = -\frac{b}{2a} \) applies only to quadratic functions and not to cubic functions.

(d) Teachers should explain that the gradient of a tangent is the derivative of a function at the given point. Learners should be exposed to higher-order thinking questions and interpretation of graphs. Initially teachers should assist learners to understand what is being asked, what it looks like in the picture and which \( x \)-values are relevant to the interval of the required solution.

(e) Teachers should explain the concepts of maxima and minima and demonstrate to learners where the graph is increasing, turning or decreasing with the aid of diagrams. Software, like Geometry Sketch Pad, Graph and GeoGebra, can be useful to demonstrate the above.

QUESTION 8: APPLICATION OF CALCULUS

Common Errors and Misconceptions

(a) Many candidates could not set up the expression for the height of the container in Q8.1.

(b) In Q8.1.2 many candidates could not determine the expression for the volume of the container.

(c) Most candidates did not realise that the word ‘hence’ in Q8.3 meant that they needed to use the information given in Q8.2. Some candidates did not use the derivative to calculate the value of \( x \). Other candidates did not equate the derivative to 0.

(d) In Q8.4 many candidates substituted the values of \( x \) in the derivative function instead of the function that represented the volume of the container. Some candidates substituted \( x = 0 \) to calculate the maximum volume.

Suggestions for Improvement

(a) Teachers should expose learners to different problem-solving techniques. Teachers need to teach learners that using brackets is important when substituting a value in an expression containing more than one term. This is essential when multiplication is yet to be done.

(b) Learners need to be taught that when the question asks to ‘Show/Prove that …’, it means calculate (justify by means of mathematically correct steps) what is given and the final answer reached must match what is stated in the question. Learners need to be taught the formulae of different shapes.
Teachers must explain to learners that the word ‘hence’ means to use the information obtained in the previous question to solve the question at hand. For optimisation problems, teachers need to emphasise to learners that the first step is to differentiate, equate the derivative to 0 and then factorise or use a formula to find the value of \( x \).

Learners should be made aware that to calculate the maximum total volume, one needs to substitute the value of \( x \) obtained from equating the derivative function to zero, in the expression for the volume.

**QUESTION 9: INTEGRATION**

**Common Errors and Misconceptions**

(a) Some candidates omitted C on indefinite integrals calculations in Q9.1.1 and 9.1.2. After integrating, they still wrote down the integral notation. Generally, many candidates wrote notations incorrectly in Q9.

(b) Some candidates could not simplify the given expression correctly in Q 9.1.2.

(c) In Q9.2 candidates swapped the lower and upper limits of definite integrals and did not consider the signs. Some candidates substituted the boundaries in the given function without first integrating.

**Suggestions for Improvement**

(a) Teachers should explain that when determining indefinite integrals, C must always be added. Further, the correct use of notation should be emphasised.

(b) Simplification of expressions covered in previous grades should be revised.

(c) Teachers should indicate to learners that in definite integrals, the upper limit is always greater than the lower limit. Learners should be made aware that they need to subtract the expression of the lower limit from the expression of the upper limit. Since these expressions may involve more than one term, they need to be careful with the signs. To prevent writing incorrect signs, it is advisable that learners keep the two expressions that are to be subtracted, in brackets.
2.5 ANALYSIS OF LEARNER PERFORMANCE IN EACH QUESTION IN PAPER 2

(a) Candidates performed relatively well in Q1 and Q2. These questions were based on Analytical Geometry. Candidates performed extremely well in questions involving length and midpoint. These concepts were covered in Grades 10 and 11.

(b) Candidates performed fairly well in Q6 and Q7. These questions were based on trigonometric 2D figures.

(c) Candidates performed poorly in Q3, Q4, Q5, Q8, Q10, Q11 with Q9 being the worst answered question. These questions largely assessed work covered in Grade 12.

(d) Candidates did not adhere to the instructions stipulated in the question paper.

(e) Many candidates did not attempt the higher-order questions.

2.6 DIAGNOSTIC QUESTION ANALYSIS FOR PAPER 2

The following graph is based on data from a random sample of candidates. While this graph might not accurately reflect national averages, it is useful to assess the relative degrees of challenge of each question as experienced by candidates.

Graph 2.6.1 Average Percentage Performance per Question for Paper 2
Figure 2.6.1 Average Percentage Performance per Question for Paper 2

Figure 2.6.2 Average Percentage Performance per Subquestion for Paper 2

2.7 ANALYSIS OF LEARNER PERFORMANCE IN EACH QUESTION IN PAPER 2

QUESTION 1: ANALYTICAL GEOMETRY

Common Errors and Misconceptions

(a) When answering Q1.1, candidates failed to take into account that FD was parallel to the x-axis. They also did not realise that the question integrated Euclidean geometry and assumed that $\alpha = 76^\circ$ instead of $180^\circ - 76^\circ = 104^\circ$.

(b) When answering Q1.2, some candidates made incorrect substitutions in the distance formula and some did not follow the instruction to write their answer in simplified surd form.
(c) In Q1.3 candidates calculated the gradient by using the formula \( m = \frac{y_2 - y_1}{x_2 - x_1} \) even though the coordinates of both points were not known.

(d) Some candidates, when answering Q1.4, made incorrect substitutions in the correct midpoint formula. Substitution of negative values was a challenge.

(e) In attempting to determine the equation of the perpendicular line, some candidates used the gradient of AF and not that of the perpendicular. Also, some candidates used the coordinates of A instead of the coordinates of the midpoint, even though A was not a point on the perpendicular line.

Suggestions for Improvement

(a) Learners should be taught to always read questions before answering. For each question they should establish what is required, what information is given and what information needs to be calculated.

(b) Teachers should emphasize the minimum requirements for the use of a certain formula. In this regard, attention should be paid as to which formula should be used to calculate the gradient of a straight line when two points on the straight line are given and which should be used when given the inclination of the line.

(c) Teachers should emphasize the use of brackets when substituting negative values. This will reduce confusion between the negative sign in a formula and the negative sign of the coordinate.

(d) Learners should be made aware of certain terminology. For example, when dealing with a perpendicular bisector, learners should know that the perpendicular bisector passes through the midpoint of another line segment and is perpendicular to that line segment. Learners need to be reminded that the product of the gradients of two lines that are perpendicular to each other is \(-1\), i.e. \( m_{\text{perp}} \times m_{\text{perp}} = -1 \).

QUESTION 2: ANALYTICAL GEOMETRY

Common Errors and Misconceptions

(a) Most candidates were able to calculate the value of \( r \) but they did not substitute it in the equation of the circle.

(b) Some candidates were unable to use transformation to determine the coordinates of B. Some candidates did not realise that the two tangents are parallel and hence could not use the fact that the gradient of MN = the gradient of PQ.

(c) In Q 2.1.3 most candidates could not combine the condition for parallel lines and the fact that the tangent is perpendicular to the diameter.

(d) In Q 2.2.2 some candidates were unable to determine the minor and major axes and hence they drew a graph that was not related to the equation.
Suggestions for Improvement

(a) Learners should be taught that if they are required to determine the equation of a function or relation, they should calculate the values of the unknowns (parameters). However, learners must then substitute these calculated values in the standard form of the equation of the function.

(b) Teachers should teach transformations that are implied by prescribed content, e.g. reflection of a point on a circle having the origin as centre.

(c) Teachers should always integrate different topics in their teaching. Concepts from Euclidean Geometry cannot be ignored when teaching Analytical Geometry.

(d) Teachers should teach learners the standard form of the ellipse and different ways to express the equation. Determining the major and minor axes is also important when drawing the correct shape of an ellipse.

QUESTION 3: TRIGONOMETRY

Common Errors and Misconceptions

(a) In Q3.1.1 some candidates calculated \((\sin 3^\circ) \times (32)\) instead of \(\sin(3 \times 32^\circ)\). This arose from the incorrect use of brackets when candidates entered the information into the calculator. Some made these calculations while their calculators were set in Radians mode. They obtained incorrect answers.

(b) Most candidates were unable to manipulate inverse trigonometric ratios. They were unable to use their calculators to determine the numerical value of \(\frac{\sec^2 \theta - 1}{\tan \alpha}\).

(c) In Q3.2.1 candidates used calculators to evaluate \(\sin 35^\circ\), against the instruction given. This answer was not awarded any marks as it was not in terms of \(m\).

(d) In Q3.2.2 some candidates calculated the numerical value of \(\left(\cos \frac{29 \pi}{36} \right) \left(\tan \frac{7 \pi}{36}\right)\) with their calculators in Degrees mode instead of Radians mode.

(e) In Q3.3.1 many candidates transposed the 2 and incorrectly wrote the statement as \(\cos \theta + \sin \theta = -2\). Some candidates just wrote down what they were expected to show.

(f) Most candidates were unable to determine the reference angle. Some wrote the reference angle as \(-63.43^\circ\) and not as a positive angle in the 1st quadrant. Some did not consider the quadrant where the tangent ratio is negative.
Suggestions for Improvement

(a) Learners should be taught how to use a calculator and to always check the mode before making calculations. The mode selected on the calculator and angle units should be the same.

(b) Teachers should revise the inverse trigonometric ratios covered in Grade 10 and how to manipulate them so that learners are able to use a calculator to calculate the value of an expression.

(c) Teachers should emphasise the importance of following the instructions in a question to learners.

(d) Teachers should expose learners to applications of Pythagoras’ theorem in which variables are used.

(e) Learners must be taught how to convert angle measures from degrees to radians and from radians to degrees.

(f) Learners should be taught that the reference angle is always in the 1st quadrant, i.e. it is always a positive acute angle. They should be taught the quadrants where specific trigonometric ratios are positive or negative.

QUESTION 4: TRIGONOMETRY

Common Errors and Misconceptions

(a) In Q 4.1.1 some candidates wrote the answer as 1 instead of −1.

(b) Simplification of trigonometric expressions using identities proved to be a challenge in Q4.1.2.

(c) Candidates were unable to use a calculator to calculate $\sec 60^\circ$.

(d) In Q4.2.2 candidates were unable to establish the correct quadrant for the given trigonometric ratio. This resulted in them writing the incorrect sign when they did the reductions.

Suggestions for Improvement

(a) A good understanding of Trigonometry is dependent on learners having knowledge of reduction formulae and fundamental identities. If learners are unsure about the identities, then they must refer to the formula sheet.

(b) Proficiency in answering questions that involve reductions and identities increase through practice. Teachers should expose learners to a variety of problems that involve identities.

(c) Teachers should revise inverse trigonometric ratios done in Grade 10 and ways of manipulating them to calculate their numerical value using a calculator. Learners should be given more work that requires calculations of numerical values of inverse trigonometric ratios.

(d) Teachers should emphasise to learners in which quadrants the specific ratios are positive or negative.
QUESTION 5: TRIGONOMETRIC FUNCTIONS

Common Errors and Misconceptions

(a) Candidates were unable to differentiate between the concepts of ‘period’ and ‘domain’.

(b) Candidates were unable to calculate the value of $b$ because they could not relate the period of the graph to the value of $b$.

(c) In Q5.3 most candidates added $21.5^\circ$ to $135^\circ$ instead of subtracting it from $180^\circ$.

(d) Candidates were unable to interpret graphs.

Suggestions for Improvement

(a) The teaching of trigonometric functions extends beyond the sketching of graphs. Teachers should also explain the characteristics of each basic trigonometric function and how these characteristics change when the basic graph is transformed.

(b) The purpose of reduction formulae must be explained. It is important to relate reduction formulae to the CAST diagram. Once learners begin to understand how to reduce trigonometric ratios with larger angles and trigonometric ratios with an acute angle, they are more likely to get the reduction correct. As a starting point, learners should be able to identify the quadrant in which the original angle lies.

(c) Interpretation of graphs should be taught thoroughly and should be included in both informal and formal assessment tasks so that candidates are well prepared to answer similar questions in the NSC examinations.

QUESTION 6: TRIGONOMETRY

Common Errors and Misconceptions

(a) Many candidates were unable to make AC the subject of the formula in Q 6.1.

(b) Some candidates were not able to apply the area rule and they were unable to make $\beta$ the subject of the formula.

(c) In Q6.3 candidates made incorrect substitutions in the cosine formula.
Suggestions for Improvement

(a) Revision should be done of the sine rule, cosine rule and area rule with many examples that involve changing subject of the formula.

(b) A structured approach to teaching 3-D problems should be adopted. Firstly, learners need to identify the triangles in the sketch. Then they need to establish what information is given about each triangle. Lastly, they need to relate the question to one of the triangles in the sketch. If the triangle has sufficient information to respond to the question, then the solution is a direct one. If there is insufficient information in the triangle, learners may have to look at the other triangles for clues to calculate the missing side(s) and/or angle(s). A good knowledge of the minimum requirements for the application of the sine, cosine and area formulae will make this task easier.

(c) Both two- and three-dimensional figures must be tested through informal and formal assessment tasks.

QUESTION 7: EUCLIDEAN GEOMETRY

Common Errors and Misconceptions

(a) Candidates could not recall the statement of the theorem in Q7.1.

(b) Candidates wrote incorrect reasons or incomplete reasons for the correct magnitude of angles.

(c) Candidates struggled to work with the variable in Q7.2.2, i.e. the length $x$.

Suggestions for Improvement

(a) During the teaching of Euclidean geometry, teachers should regularly give learners activities on the stating of theorems.

(b) The ‘accepted reasons’ as stated in the Examination Guidelines must be consolidated and reinforced when teaching Euclidean geometry. Teachers should cover the basic work thoroughly. An explanation of the theorem should be accompanied by showing the relationship in a diagram.

(c) Learners should also be exposed to calculating the lengths of the sides, not only the sizes of angles.
QUESTION 8: EUCLIDEAN GEOMETRY

Common Errors and Misconceptions

(a) Candidates struggled with the application of circle geometry theorems.

(b) There was often confusion as to when the theorem could be used as a reason and when the converse would apply.

Suggestions for Improvement

(a) Teachers should expose candidates to questions that require different ways of applying circle geometry theorems.

(b) Teachers should explain the difference between a theorem and its converse. Application of converse theorems should be known and practised in solving riders from lower grades.

(c) Learners should be exposed to questions that have a mixture of theorems, i.e. questions should include knowledge of angles in circles, cyclic quadrilaterals and tangents to the circle.

QUESTION 9: EUCLIDEAN GEOMETRY

Common Errors and Misconceptions

(a) Some candidates were unable to apply the tangent theorems in Q9.2.1.

(b) Some candidates struggled to apply similarity theorems. In Q9.2.2 most candidates assumed that E is the midpoint OG.

Suggestions for Improvement

(a) Teachers should expose candidates to different ways of applying circle geometry theorems.

(b) Learners should be exposed to questions that assess the integration of circle theorems and similarity theorems.
QUESTION 10: CIRCLES, ANGLES AND ANGULAR MOVEMENT

Common Errors and Misconceptions

(a) Some candidates did not realise that BC = OC − OB = 20 − 1.5 = 18.5 cm.

(b) Some candidates were able to make the correct substitution in the application of Pythagoras’ theorem but failed to factorise the resulting quadratic equation in Q10.1.2. Many candidates failed to recall that in order to use Pythagoras’ theorem, two sides of the right-angled triangle must be known.

(c) In Q10.2.1 candidates struggled to determine the size of \( \hat{A}OB \) in degrees. Conversion from degrees to radians was also a challenge for some candidates.

(d) Some candidates used the correct formula \( s = r \theta \) but used the angle measured in degrees instead of radians.

(e) In Q10.2.2 many candidates wrote the correct formula

\[
\text{Area of a sector} = \frac{r s}{2} = \frac{r^2 \theta}{2}
\]

but substituted incorrectly. Once again, they used the angle in degrees instead of in radians.

Suggestions for Improvement

(a) Teachers should inform learners that skills are transferable from one topic to another in Mathematics. Certain algebraic skills may well be required in Trigonometry and Euclidean Geometry.

(b) Learners should be advised to read the given information with understanding.

(c) Teachers need to revise the calculation of arc length done in Grade 11. Learners should also be exposed to problem-solving strategies so that they will be able to solve real-life scenarios that require application of formulae. It should be emphasised that the angles used in the formulae \( s = r \theta \) and

\[
\text{Area of a sector} = \frac{r s}{2} = \frac{r^2 \theta}{2}
\]

should be in radians.
QUESTION 11: MENSURATION

Common Errors and Misconceptions

(a) In Q11.1.1 some candidates did not read the instruction and left the answer as $\sqrt{2}$.

(b) In Q11.1.2 many candidates did not use two thirds and most did not make the correct substitution in the formula.

(c) Many candidates used the formulae given and did not use the volume given.

(d) Some candidates wrote the diameter and did not realise that they must divide by 2.

(e) Many candidates did not adapt the formulae for surface areas and hence they got incorrect values.

Suggestions for Improvement

(a) Teachers need to allow learners to use the formula sheet in informal and formal assessments in Grade 12.

(b) Mensuration should be thoroughly revised and assessed from lower grades. Learners should know how to adapt a formula in relation to a given situation.